Lecture 7 FCS, Autocorrelation, PCH, Cross-correlation

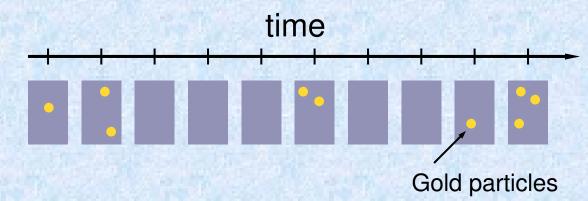
Joachim Mueller

Principles of Fluorescence Techniques Laboratory for Fluorescence Dynamics

Figure and slide acknowledgements: Enrico Gratton

Historic Experiment: 1st Application of Correlation Spectroscopy

(Svedberg & Inouye, 1911) Occupancy Fluctuation

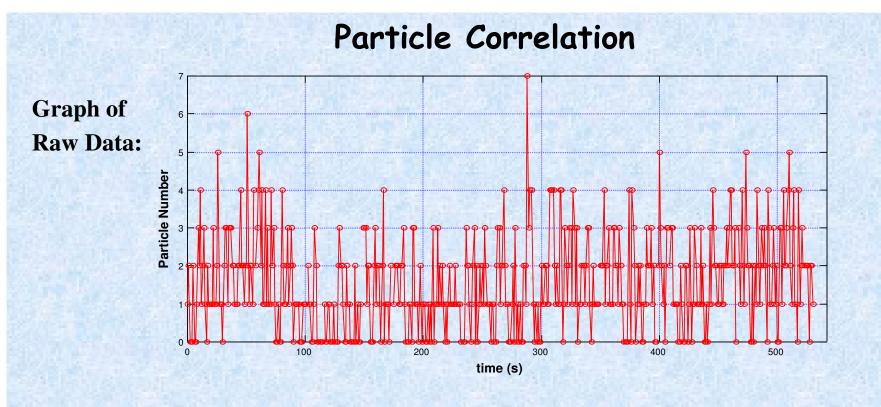


 $120002001324123102111131125111023313332211122422122612214234524114131142\\ 3100100421123123201111000111_2110013200000100110001000232210021100002010\\ 01_333122000231221024011102_12221122310001103311102101100101030113121210\\ 10121111211_100032210123020121213211101100233122421100012030101002217344\\ 101010021122114444212114401321233143130112221233101211112224122311133221\\ 32110000410432012120011322231200_253212033233111100210022013011321131200\\ 101314322112211223234422230321421532200202142123232043112312003314223452\\ 13411041232220221$

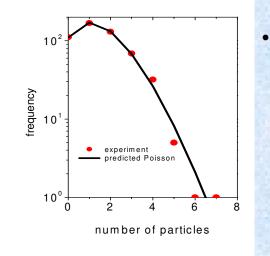
Svedberg and Inouye, Zeitschr. F. physik. Chemie 1911, 77:145

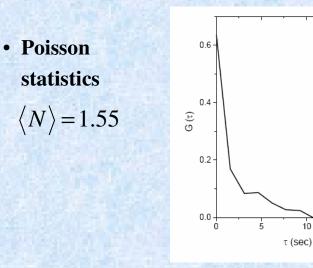
Collected data by counting (by visual inspection) the number of particles in the observation volume as a function of time using a "ultra microscope"

Statistical analysis of raw data required



*Histogram of particle counts





*Autocorrelation

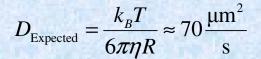
10

 Autocorrelation not available in the original paper. It can be easily calculated today.

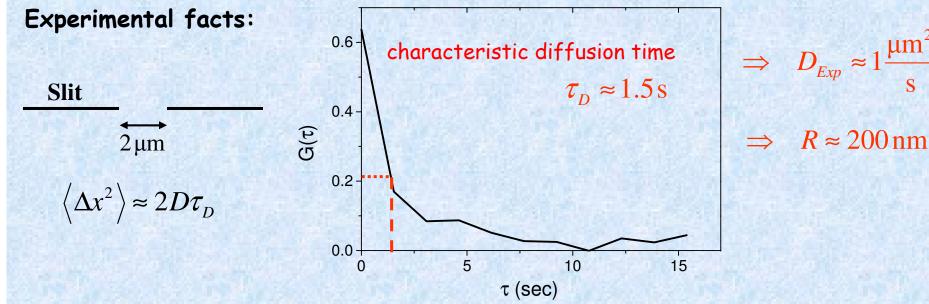
 $\langle N \rangle = \frac{1}{G(0)} = 1.56$

What we learn from the correlation function?

Svedberg claimed: Gold colloids with radius R = 3 nm



(Stokes-Einstein)



Conclusion: Bad sample preparation

The ultramicroscope was invented in 1903 (*Siedentopf and Zsigmondy*). They already concluded that scattering will not be suitable to observe single molecules, but fluorescence could.

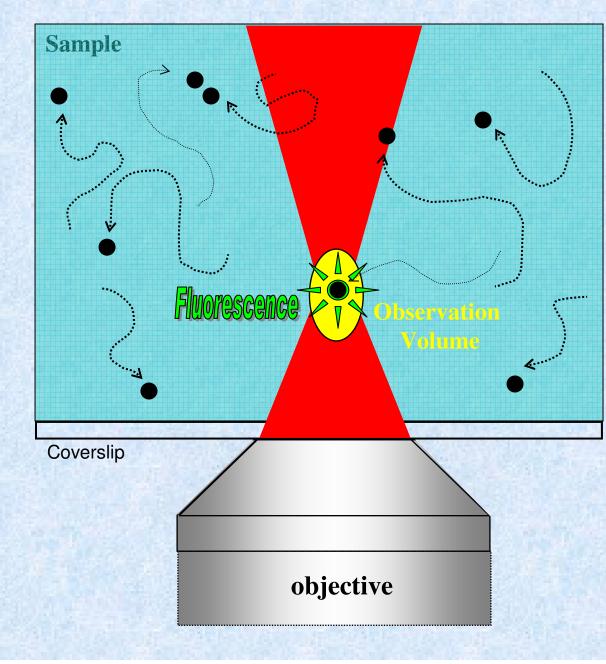
Fluorescence Correlation Spectroscopy (FCS)

In FCS Fluctuations are in the Fluorescence Signal

Diffusion Enzymatic Activity Phase Fluctuations Conformational Dynamics Rotational Motion Protein Folding

Example of processes that could generate fluctuations

Generating Fluctuations By Motion



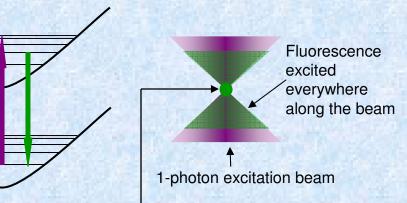
What is Observed? 1. The Rate of Motion

2. The Concentration of Particles

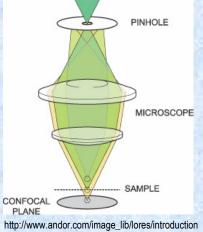
3. Changes in the Particle Fluorescence while under Observation, for example conformational transitions

Observation Volume in 1- & 2-Photon Excitation.

1-Photon:

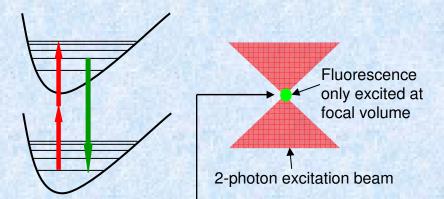


Observation Volume: Defined by the confocal pinhole size, wavelength, magnification and numerical aperture of the objective

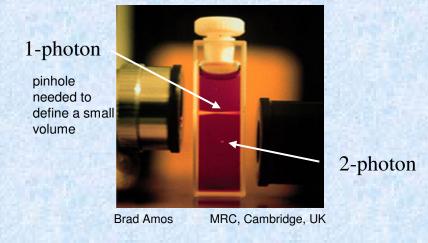


 \rightarrow

2-Photon:



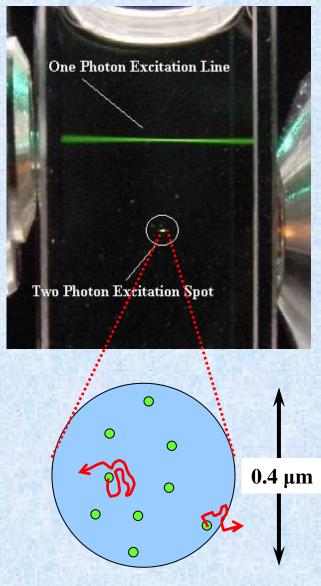
Observation Volume: Defined by wavelength, and numerical aperture of the objective



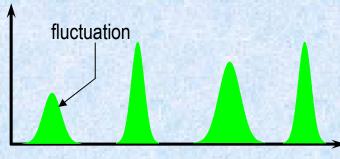
Approximately 1 µm³

Two-Photon FCS

Two-photon effect



Fluorescence signal at detector

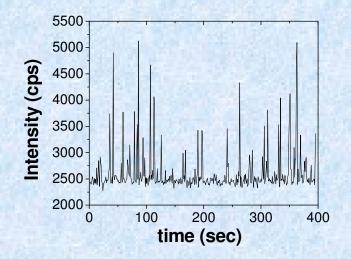


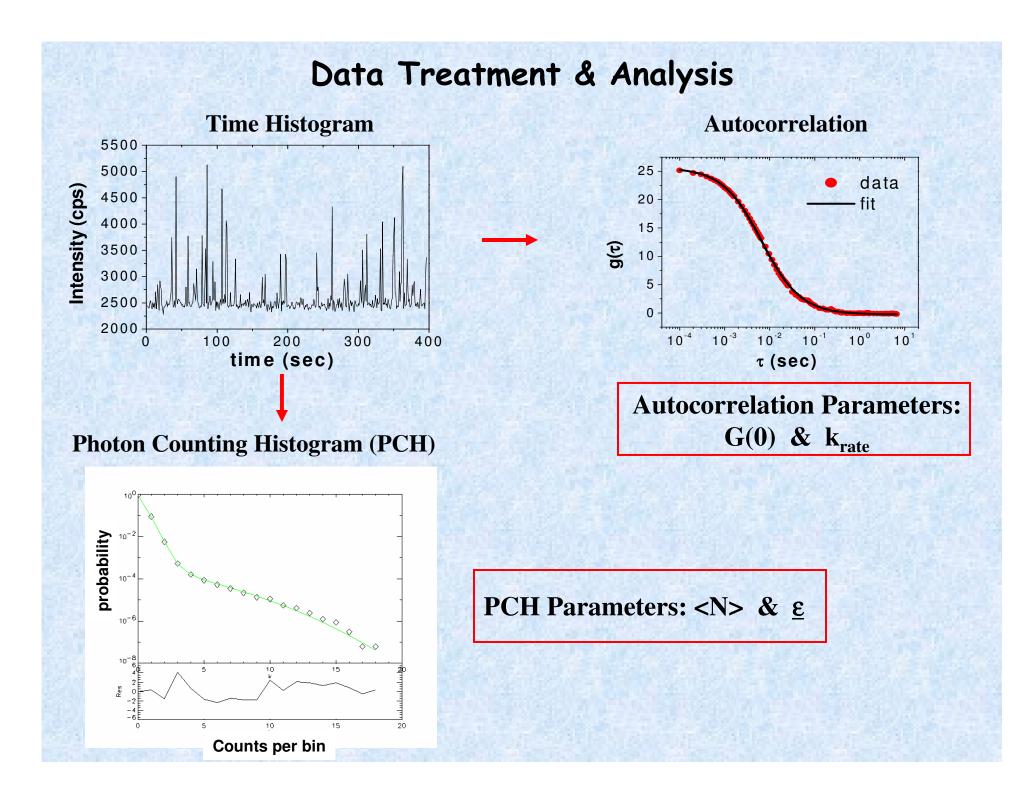
time

Example:

F

Fluorescently labeled viral particles





Autocorrelation Function

$$G(\tau) = \frac{\left\langle \delta F(t) \delta F(t+\tau) \right\rangle}{\left\langle F(t) \right\rangle^2}$$

Factors influencing the fluorescence signal:

$$F(t) = \kappa Q \int d\mathbf{r} W(\mathbf{r}) C(\mathbf{r}, t)$$

 κQ = quantum yield and detector sensitivity (how bright is our probe). This term could contain the fluctuation of the fluorescence intensity due to internal processes

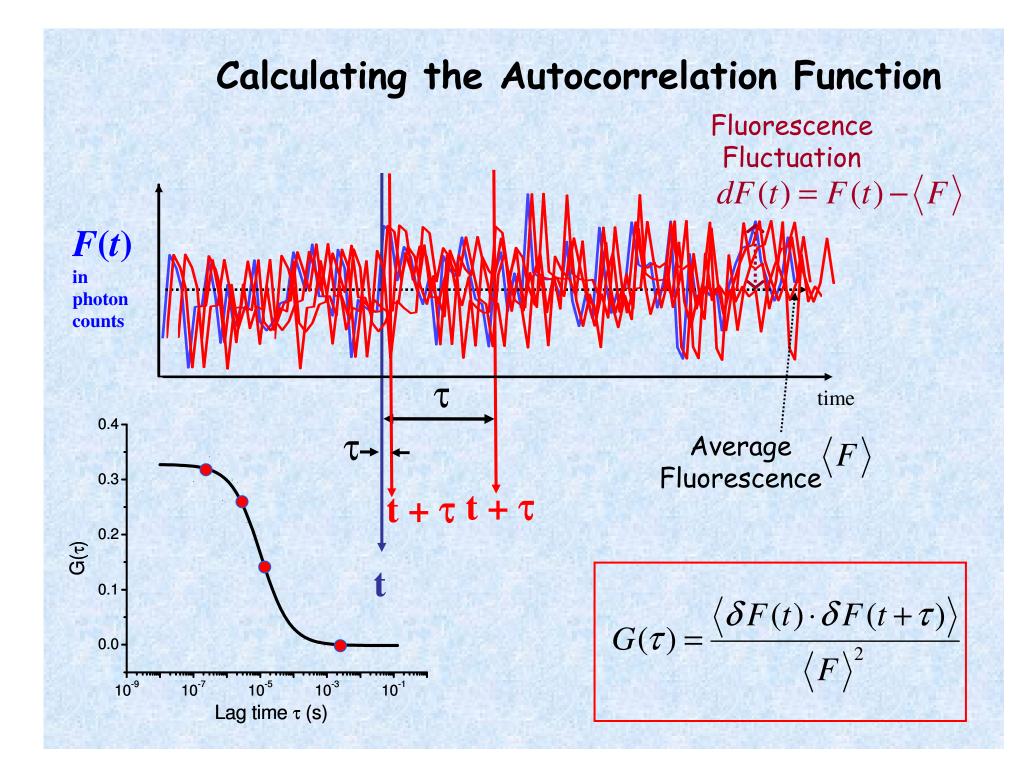
W(r) describes our observation volume

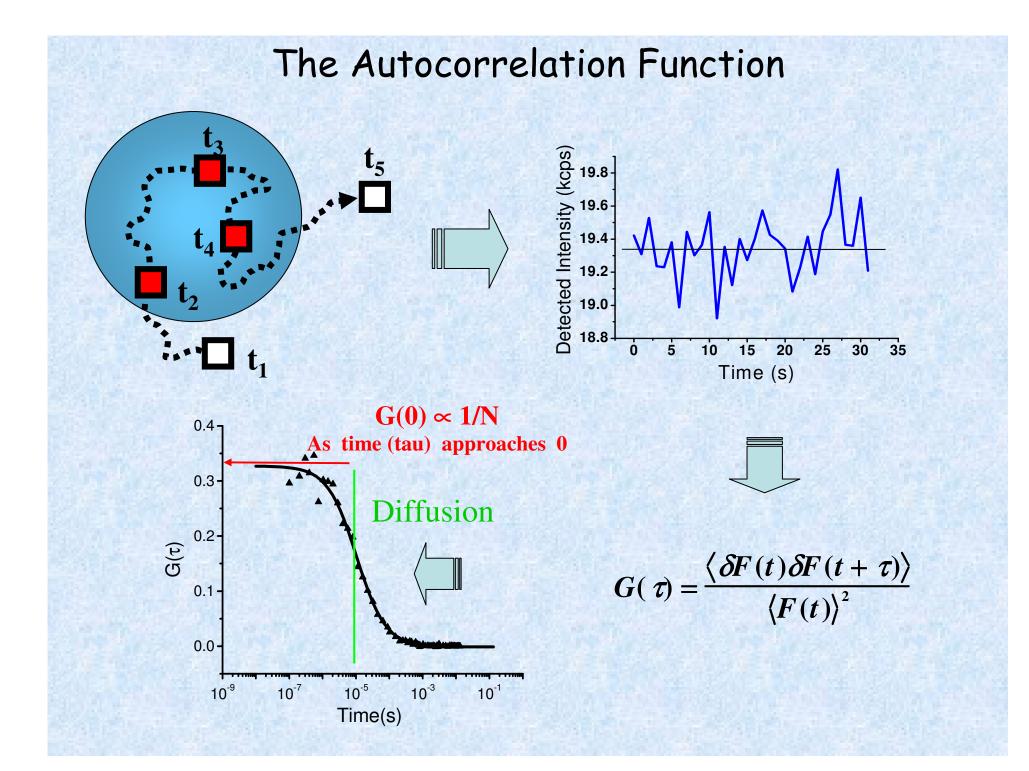
C(r,t) is a function of the fluorophore concentration over time. This is the term that contains the "physics" of the diffusion processes

Average fluorescence signal: $\langle F(t) \rangle$

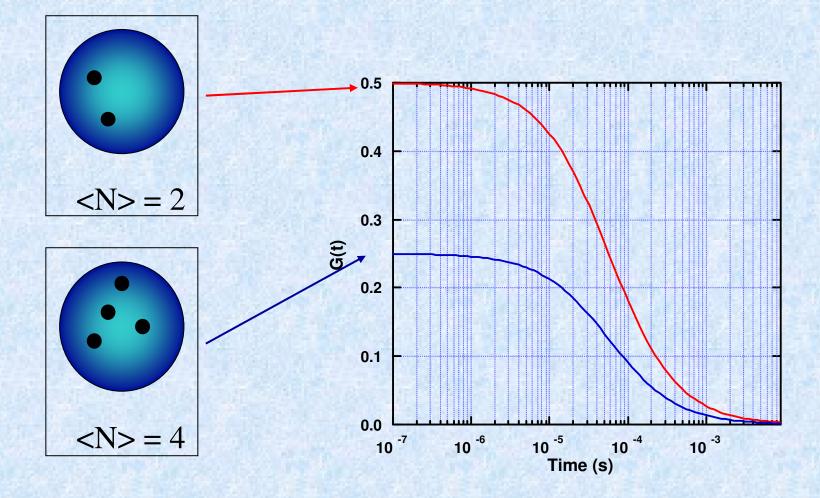
Fluorescence fluctuation:

 $\delta F(t) = F(t) - \langle F(t) \rangle$



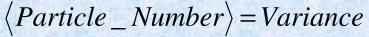


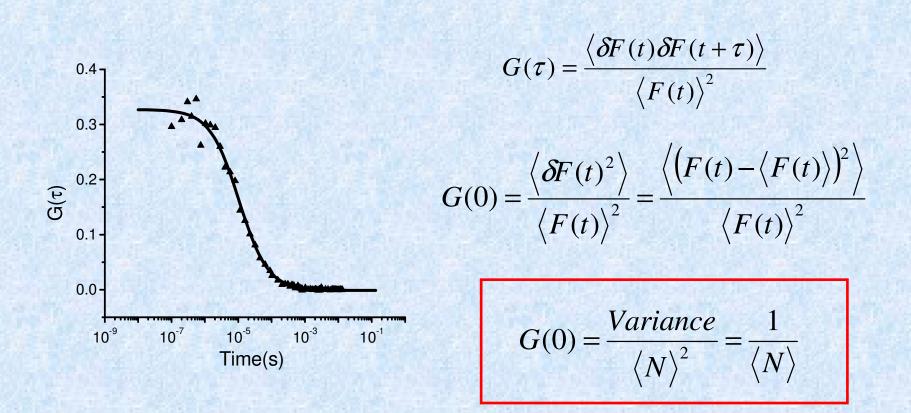
The Effects of Particle Concentration on the Autocorrelation Curve

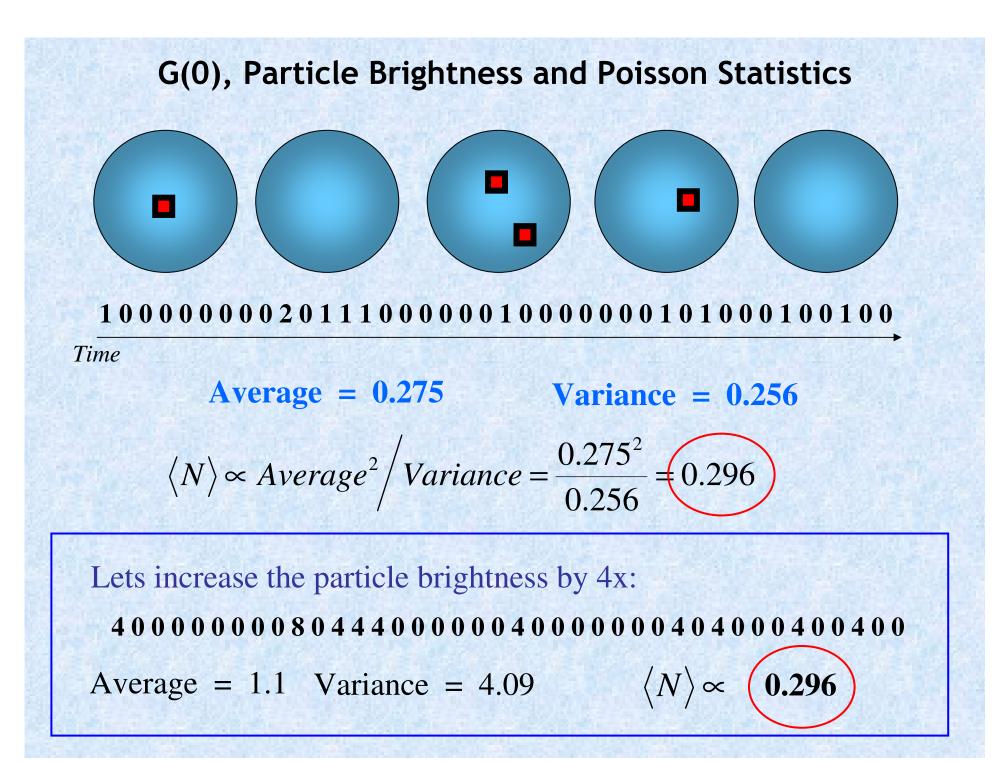


Why Is G(0) Proportional to 1/Particle Number?

A Poisson distribution describes the statistics of particle occupancy fluctuations. In a Poissonian system the variance is proportional to the average number of fluctuating species:

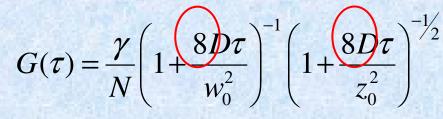




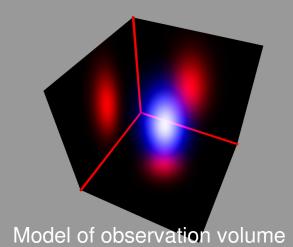


Effect of Shape on the (2-Photon) Autocorrelation Functions: (for simple diffusion)

For a 3-dimensional Gaussian excitation volume:



γ:



For a 2-dimensional Gaussian excitation volume:

1-photon equation contains a 4, instead of 8

 $G(\tau) = \frac{\gamma}{N} \left(1 + \frac{8D\tau}{w^2} \right)$

shape factor (0.354 for 3DG, 0.5 for 2DG) average number of particles inside volume N: D: Diffusion coefficient

radial beam waist of two-photon laser spot w.:

axial beam waist of two-photon laser spot Z_o:

Additional Equations:

3D Gaussian Confocor analysis:

$$G(\tau) = 1 + \frac{1}{N} \left(1 + \frac{\tau}{\tau_D} \right)^{-1} \cdot \left(1 + S^2 \cdot \frac{\tau}{\tau_D} \right)^{-1}$$

... where N is the average particle number, τ_D is the diffusion time (related to D, $\tau_D = w^2/8D$, for two photon and $\tau_D = w^2/4D$ for 1-photon excitation), and S is a shape parameter, equivalent to w/z in the previous equations.

Note: The offset of one is caused by a different definition of $G(\tau)$: $G(\tau) = \frac{\langle F(t+\tau) \cdot F(t) \rangle}{\langle F \rangle^2}$

Triplet state term:

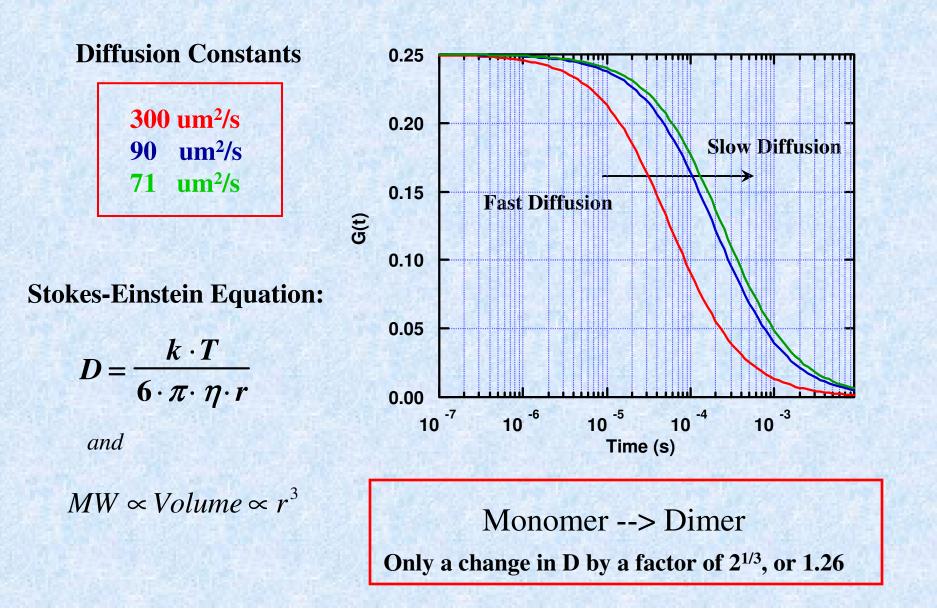
$$(1+\frac{T}{1-T}e^{\frac{-\tau}{\tau_T}})$$

...where T is the triplet state amplitude and τ_T is the triplet lifetime.

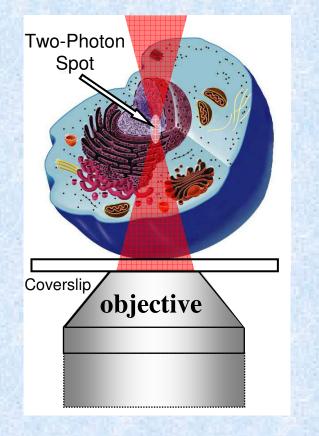
Orders of magnitude (for 10nM solution, small molecule, water)

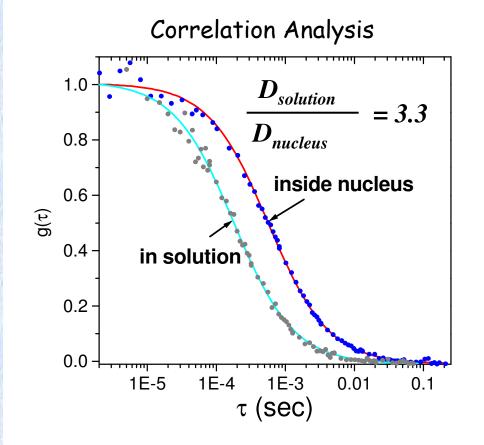
Volume	Device S	Size(µm)	Molecules	Diffusion Time (s)
milliliter	cuvette	10000	6x10 ¹²	104
microliter	plate well	1000	6x10 ⁹	10 ²
nanoliter	microfabricat	ion 100	6x10 ⁶	1
picoliter	typical cell	10	6x10 ³	10 ⁻²
femtoliter	confocal volu	me 1	6x10 ⁰	10-4
attoliter	nanofabricati	on 0.1	6x10 ⁻³	10 ⁻⁶

The Effects of Particle Size on the Autocorrelation Curve



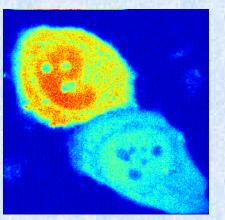
FCS inside living cells

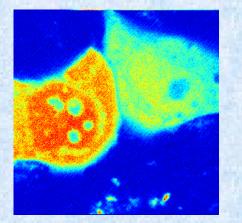




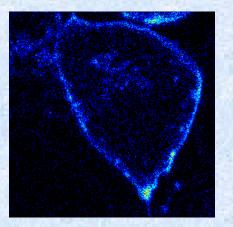
Measure the diffusion coefficient of Green Fluorescent Protein (GFP) in aqueous solution in inside the nucleus of a cell.

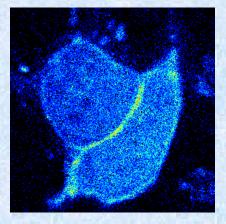
Autocorrelation Adenylate Kinase -EGFP Chimeric Protein in HeLa Cells





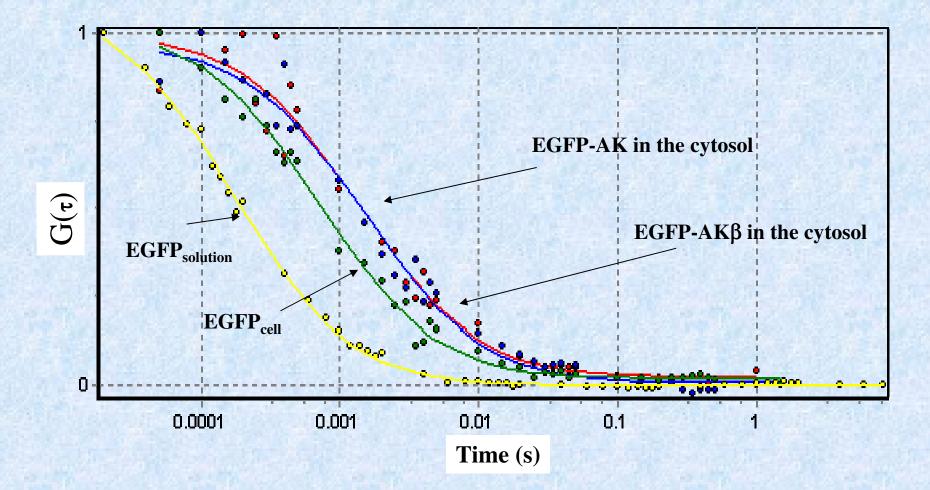
Examples of different Hela cells transfected with AK1-EGFP





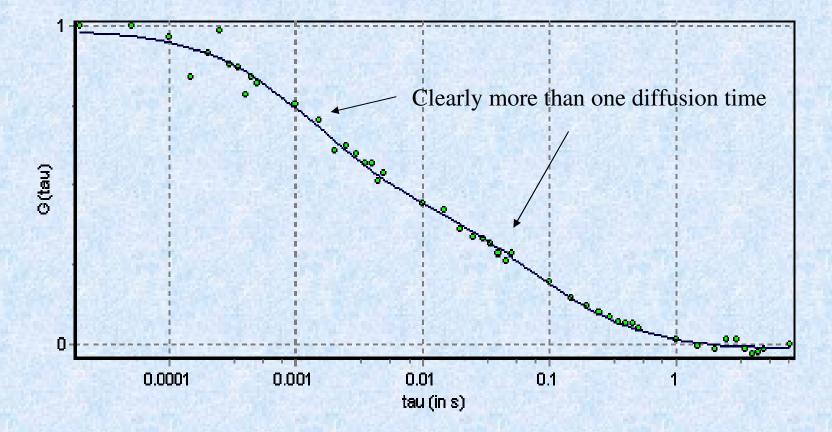
Examples of different *Hela* cells transfected with AK1β -EGFP *Qiao Qiao Ruan, Y. Chen, M. Glaser & W. Mantulin Dept. Biochem & Dept Physics- LFD Univ II, USA*

Autocorrelation of EGFP & Adenylate Kinase - EGFP



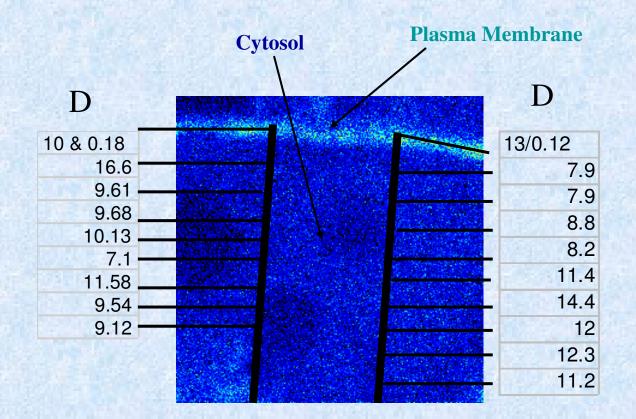
Normalized autocorrelation curve of EGFP in solution (•), EGFP in the cell (•), AK1-EGFP in the cell(•), AK1 β -EGFP in the cytoplasm of the cell(•).

Autocorrelation of Adenylate Kinase –EGFP on the <u>Membrane</u>



A mixture of AK1b-EGFP in the cytoplasm and membrane of the cell.

Autocorrelation Adenylate Kinase β -EGFP



Diffusion constants (um^2/s) of AK EGFP-AK β in the cytosol -EGFP in the cell (HeLa). At the membrane, a dual diffusion rate is calculated from FCS data. Away from the plasma membrane, single diffusion constants are found.

Multiple Species

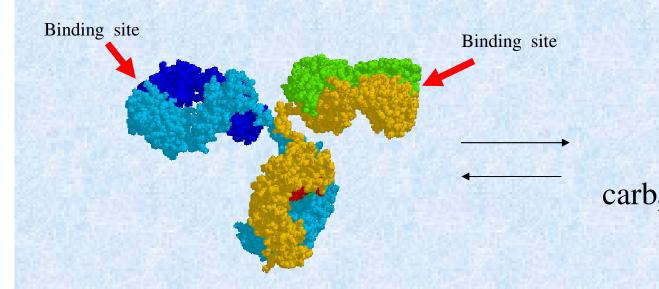
Case 1: Species differ in their diffusion constant D

Autocorrelation function can be used:

 $G(\tau)_{sample} = \sum_{i=1}^{M} f_i^2 \cdot G(0)_i \cdot \left(1 + \frac{8D\tau}{w_{2DG}^2}\right)^{-1} \quad (2D-Gaussian Shape)$ $f_i \text{ is the fractional fluorescence intensity of species } i.$ $G(0)_{sample} = \sum f_i^2 \cdot G(0)_i$

 $G(0)_{sample}$ is no longer γN !

Antibody - Hapten Interactions



Digoxin: a cardiac glycoside used to treat congestive heart failure. Digoxin competes with potassium for a binding site on an enzyme, referred to as potassium-ATPase. Digoxin inhibits the Na-K ATPase pump in the myocardial cell membrane.

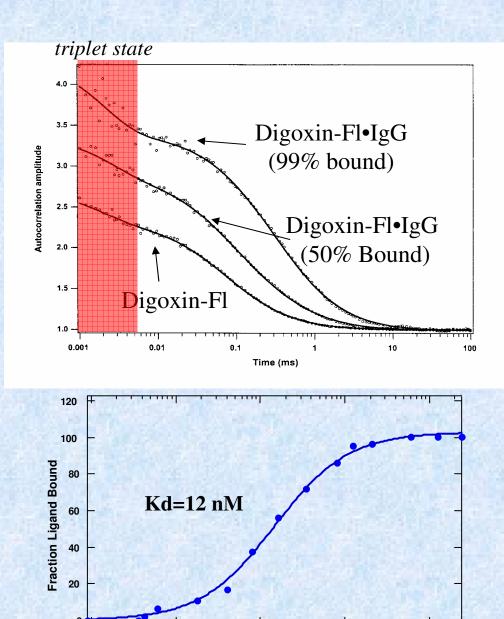
OH | CH

ĠН

Mouse IgG: The two heavy chains are shown in yellow and light blue. The two light chains are shown in green and dark blue...J.Harris, S.B.Larson, K.W.Hasel, A.McPherson, "Refined structure of an intact IgG2a monoclonal antibody", Biochemistry 36: 1581, (1997).

Anti-Digoxin Antibody (IgG) Binding to Digoxin-Fluorescein

Autocorrelation curves:



10 -8

[Antibody] _{free} (M)

10 -7

10 -6

10⁻¹⁰

10⁻⁹

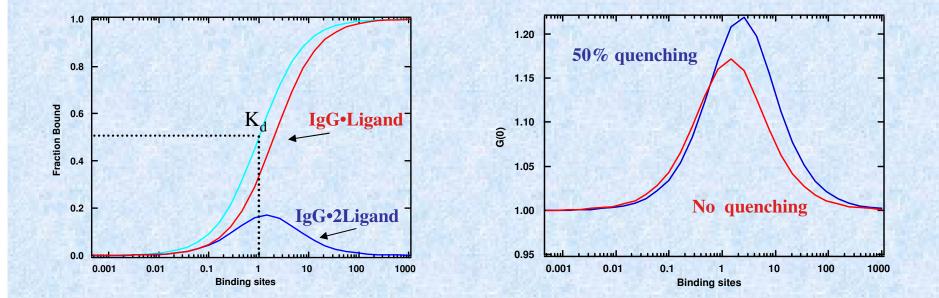
Binding titration from the autocorrelation analyses:

$$F_{b} = \frac{m \cdot S_{free}}{K_{d} + S_{free}} + c$$

S. Tetin, K. Swift, & , E, Matayoshi , 2003

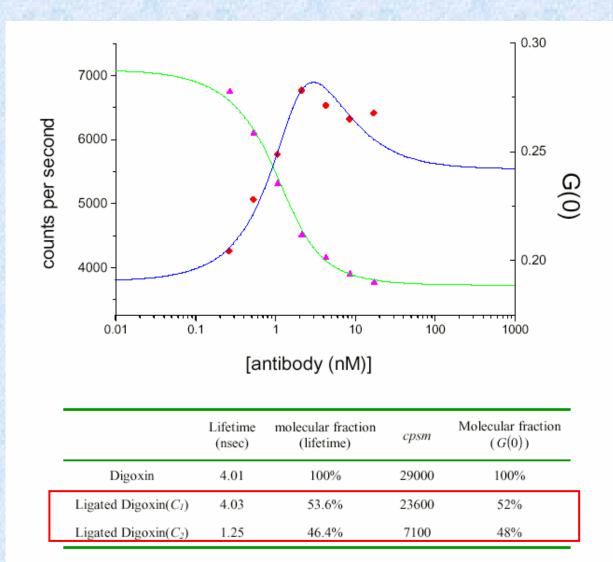
Two Binding Site Model

 $IgG + 2 Ligand-Fl \longleftarrow IgG \bullet Ligand-Fl + Ligand-Fl \longleftarrow IgG \bullet 2Ligand-Fl$



 $[Ligand]=1, G(0)=1/N, K_d=1.0$

Digoxin-FL Binding to IgG: G(0) Profile



Y. Chen, Ph.D. Dissertation; Chen et. al., <u>Biophys. J</u> (2000) 79: 1074

Multiple Species

Case 2: Species vary by a difference in brightness assuming that $D_1 \approx D_2$

The quantity G(0) becomes the only parameter to distinguish species, but we know that:

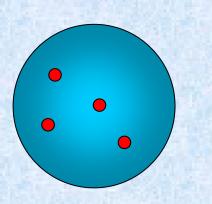
$$G(\mathbf{0})_{sample} = \sum f_i^2 \cdot G(\mathbf{0})_i$$

The autocorrelation function is not suitable for analysis of this kind of data without additional information.

We need a different type of analysis

Photon Counting Histogram (PCH)

Aim: To resolve species from differences in their molecular brightness



Poisson Distribution for particle number:

$$p(N) = \frac{\langle N \rangle^{N} \cdot e^{-\langle N \rangle}}{N!}$$

But distribution of photon counts is Non-Poissonian:

$$p(k) = PCH(\varepsilon, \langle N \rangle)$$

Single Species: p(k) is the probability of observing k photon counts

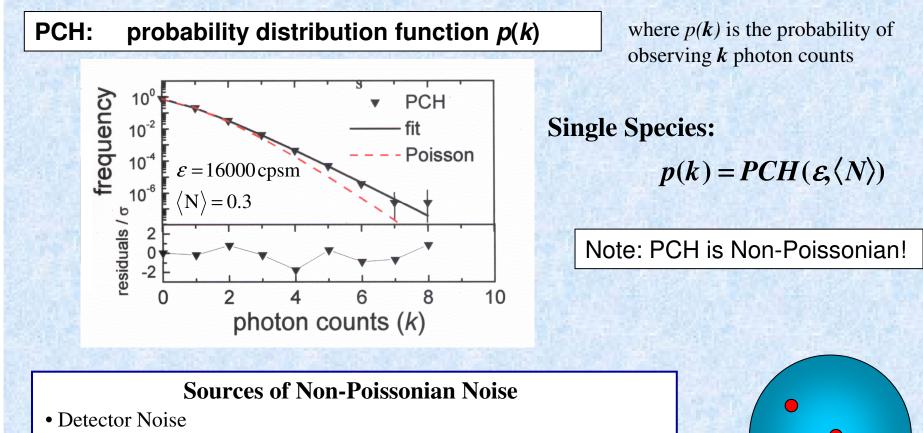
Sources of Non-Poissonian Noise

Detector Noise Diffusing Particles in an Inhomogeneous Excitation Beam* Particle Number Fluctuations* Multiple Species*

Photon Counting Histogram (PCH)

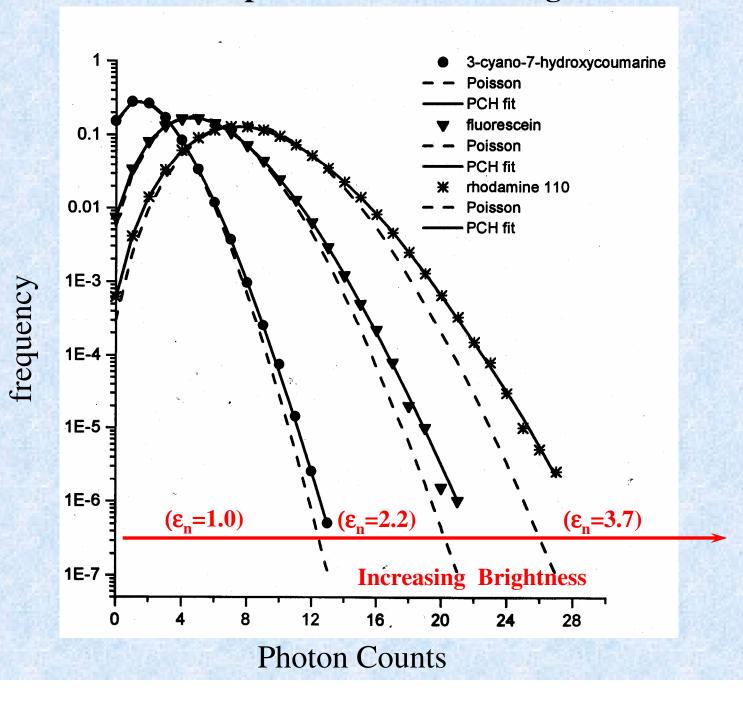
Aim: To resolve species from differences in their molecular brightness

Molecular brightness ε : The average photon count rate of a single fluorophore

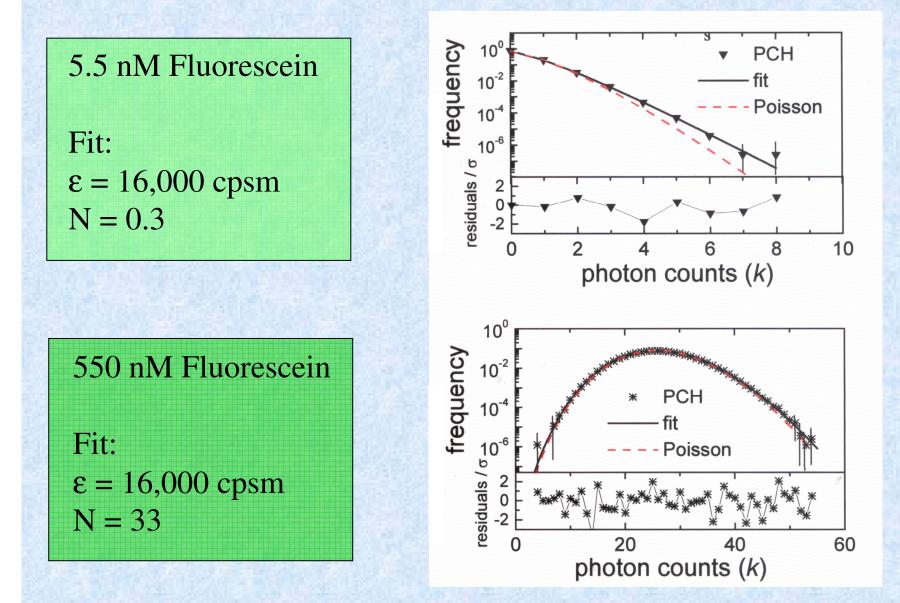


- Diffusing Particles in an Inhomogeneous Excitation Beam*
- Particle Number Fluctuations*
- Multiple Species*

PCH Example: Differences in Brightness



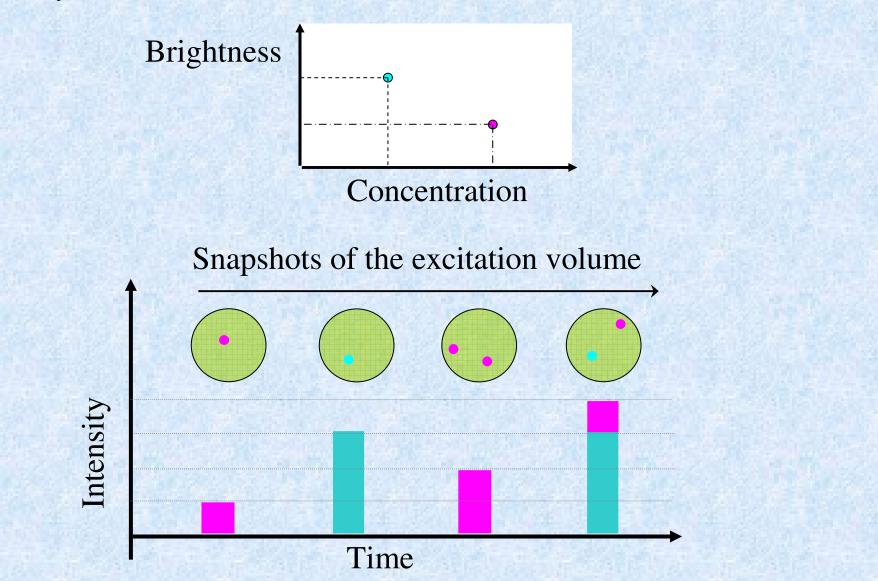
Single Species PCH: Concentration



As particle concentration increases the PCH approaches a Poisson distribution

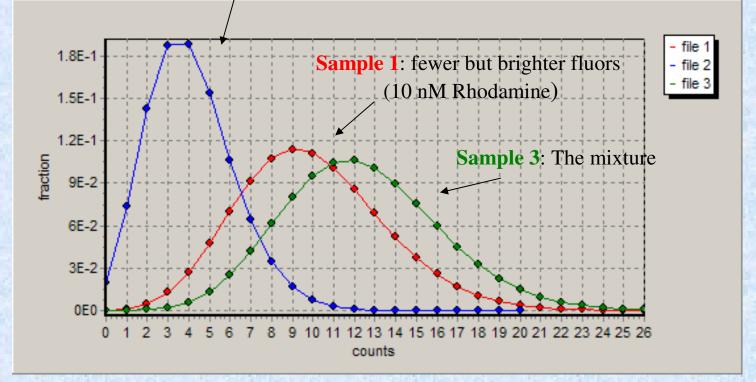
Photon Counting Histogram: Multiple Species

Binary Mixture: $p(k) = PCH(\varepsilon_1, \langle N_1 \rangle) \otimes PCH(\varepsilon_2, \langle N_2 \rangle)$



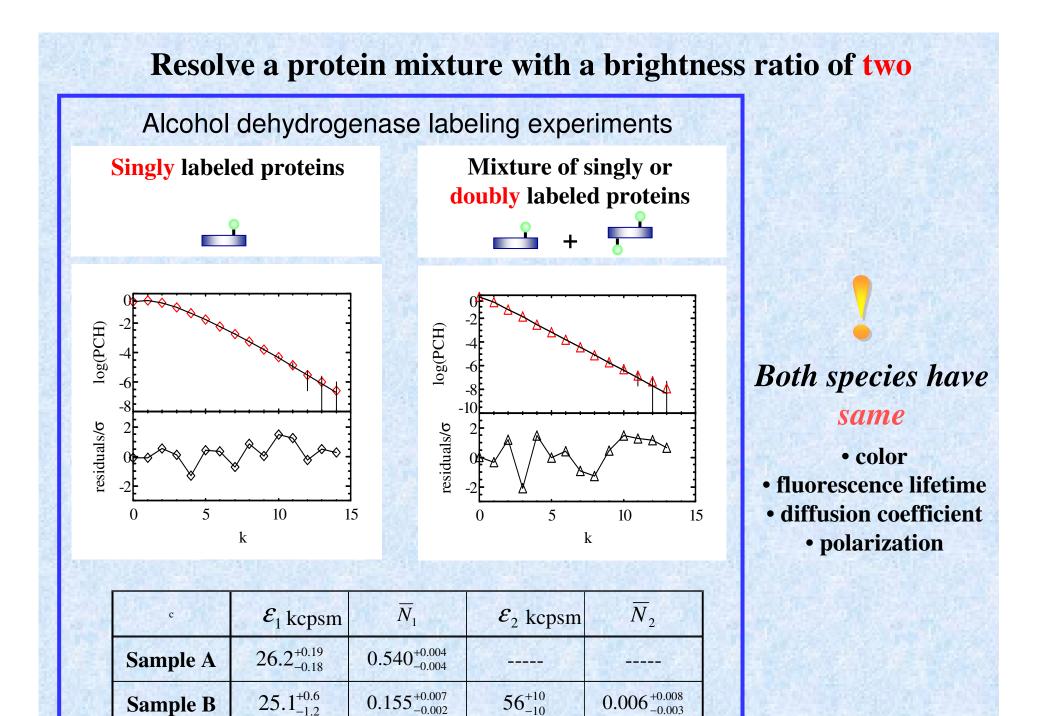
Photon Counting Histogram: Multiple Species

Sample 2: many but dim (23 nM fluorescein at pH 6.3)

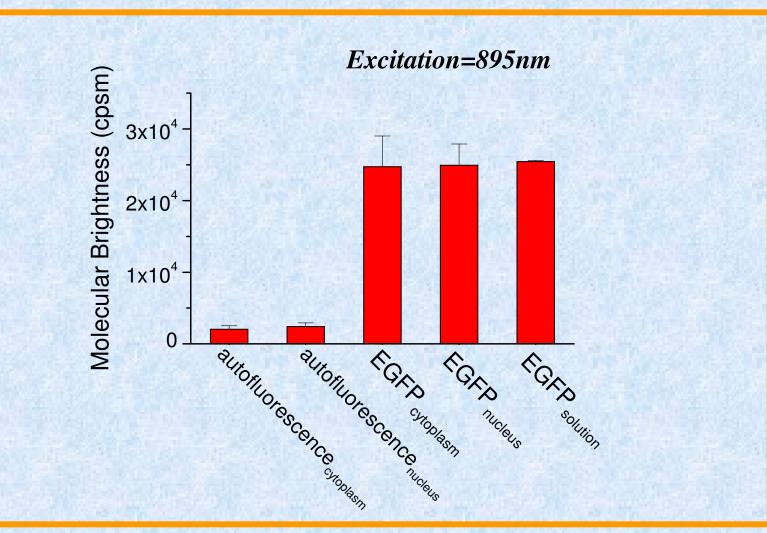


The occupancy fluctuations for each specie in the mixture becomes a convolution of the individual specie histograms. The resulting histogram is then broader than expected for a single species.

 $p(k) = PCH(\varepsilon_1, \langle N_1 \rangle) \otimes PCH(\varepsilon_2, \langle N_2 \rangle)$



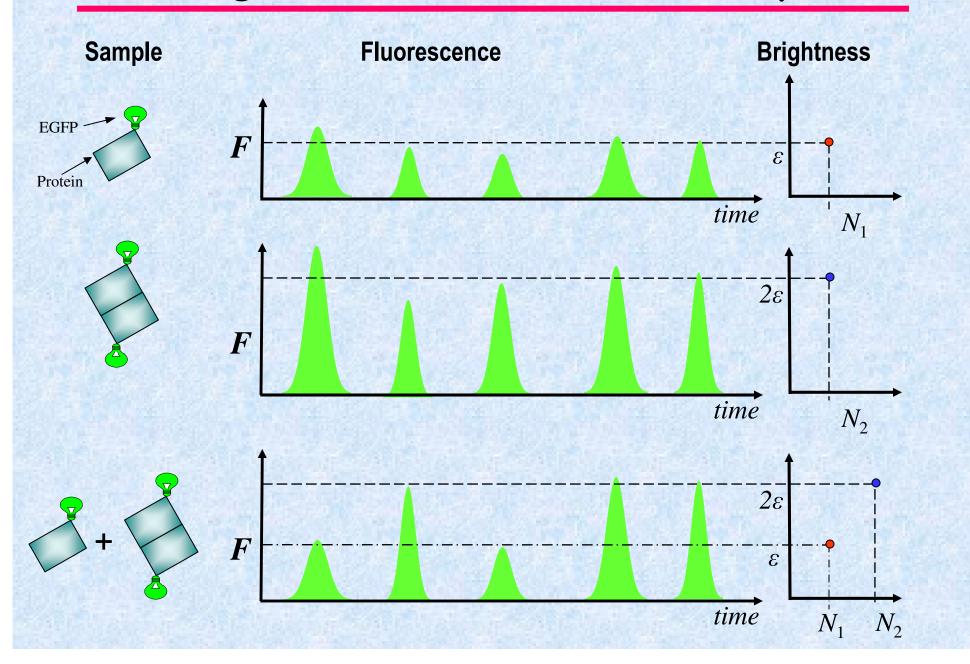
PCH in cells: Brightness of EGFP



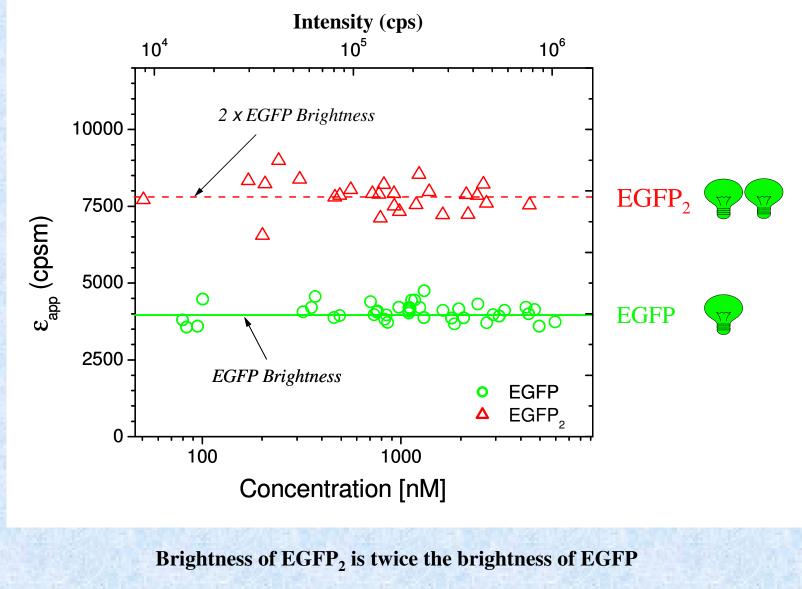
The molecular brightness of EGFP is a factor ten higher than that of the autofluorescence in HeLa cells

Chen Y, Mueller JD, Ruan Q, Gratton E (2002) Biophysical Journal, 82, 133.

Brightness Encodes Stoichiometry



Brightness and Stoichiometry



Chen Y, Wei LN, Mueller JD, PNAS (2003) 100, 15492-15497

Distinguish Homo- and Hetero-interactions in living cells

ECFP: 🕎 EYFP: 🕎

	Apparent Brightness	А	В	A B	A + B	A A
$\lambda_{2\gamma} = 905 \mathrm{nm}$	A B	3	3	2ε	3	2ε
$\lambda_{2\gamma} = 965 \mathrm{nm}$	AB	3	0	3	3	2ε

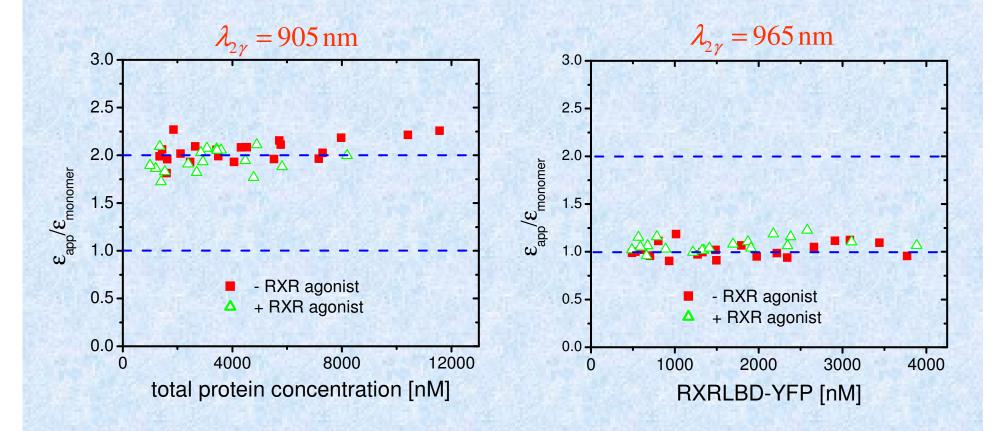
• single detection channel experiment

- distinguish between CFP and YFP by excitation (not by emission)!
- brightness of CFP and YFP is identical at 905nm (with the appropriate filters)
- you can choose conditions so that the brightness is not changed by FRET between CFP and YFP
- determine the expressed protein concentrations of each cell!

PCH analysis of a heterodimer in living cells

The nuclear receptors RAR and RXR form a tight heterodimer in vitro. We investigate their stoichiometry in the nucleus of COS cells.





Chen Y, Li-Na Wei, Mueller JD, Biophys. J., (2005) 88, 4366-4377

Two Channel Detection: Cross-correlation

Sample Excitation Volume

- 1. Isolate correlated signals.
- 2. Corrects for PMT noise

Detector 1

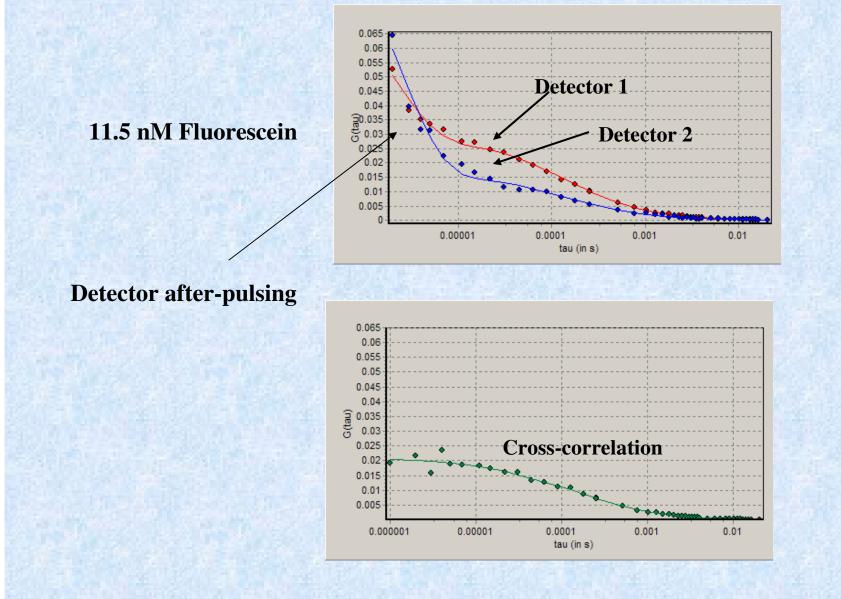
•

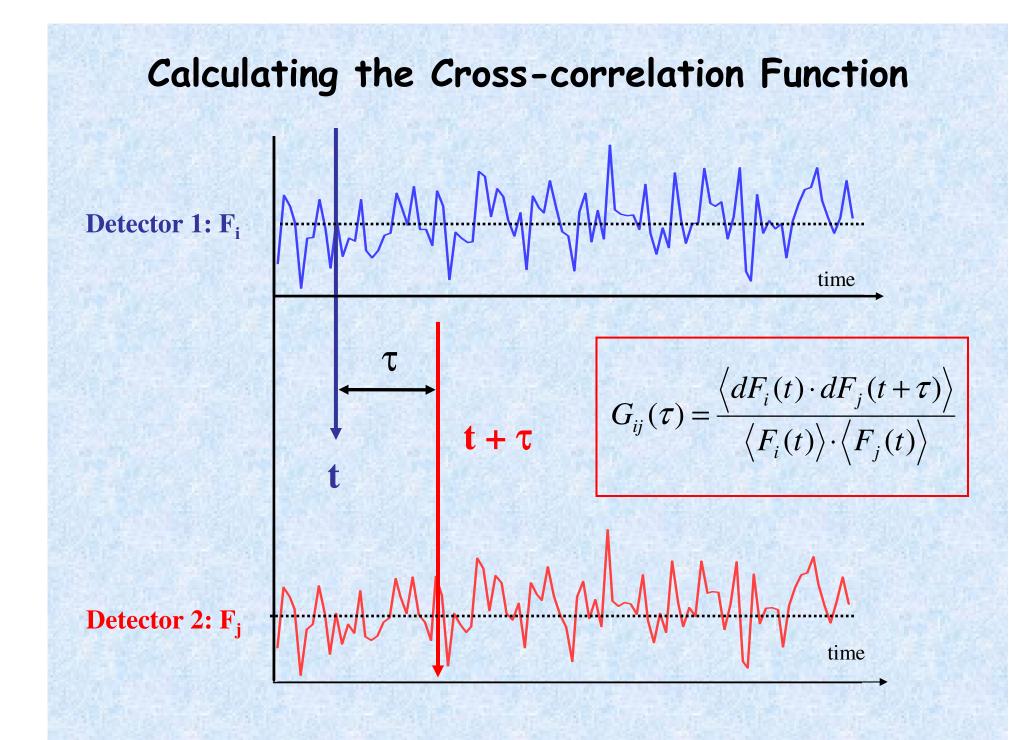
Detector 2

Beam Splitter

Each detector observes the same particles

Removal of Detector Noise by Cross-correlation





Cross-correlation Calculations

One uses the same fitting functions you would use for the standard autocorrelation curves.

Thus, for a 3-dimensional Gaussian excitation volume one uses:

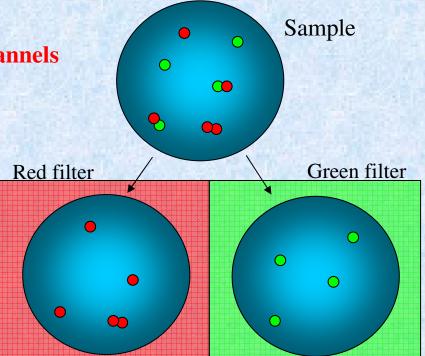
$$G_{12}(\tau) = \frac{\gamma}{N_{12}} \left(1 + \frac{8D_{12}\tau}{w^2} \right)^{-1} \left(1 + \frac{8D_{12}\tau}{z^2} \right)^{-\frac{1}{2}}$$

 G_{12} is commonly used to denote the cross-correlation and G_1 and G_2 for the autocorrelation of the individual detectors. Sometimes you will see $G_x(0)$ or C(0) used for the cross-correlation.

Two-Color Cross-correlation

The cross-correlation ONLY if particles are observed in both channels

> Each detector observes particles with a particular color



•

The cross-correlation signal:

Only the green-red molecules are observed!!

Experimental Concerns: Excitation Focusing & Emission Collection

We assume exact match of the observation volumes in our calculations which is difficult to obtain experimentally.

Excitation side:

- (1) Laser alignment
- (2) Chromatic aberration
- (3) Spherical aberration

Emission side:

- (1) Chromatic aberrations
- (2) Spherical aberrations
- (3) Improper alignment of detectors or pinhole
 - (cropping of the beam and focal point position)

Two-color Cross-correlation

Equations are similar to those for the cross correlation using a simple beam splitter:

 $\mathbf{G}_{ij}(\tau) = \frac{\left\langle \mathbf{dF}_{i}(t) \cdot \mathbf{dF}_{j}(t+\tau) \right\rangle}{\left\langle \mathbf{F}_{i}(t) \right\rangle \cdot \left\langle \mathbf{F}_{i}(t) \right\rangle}$

Information Content

Correlated signal from particles having **both colors**.

Autocorrelation from channel 1 on the green particles.

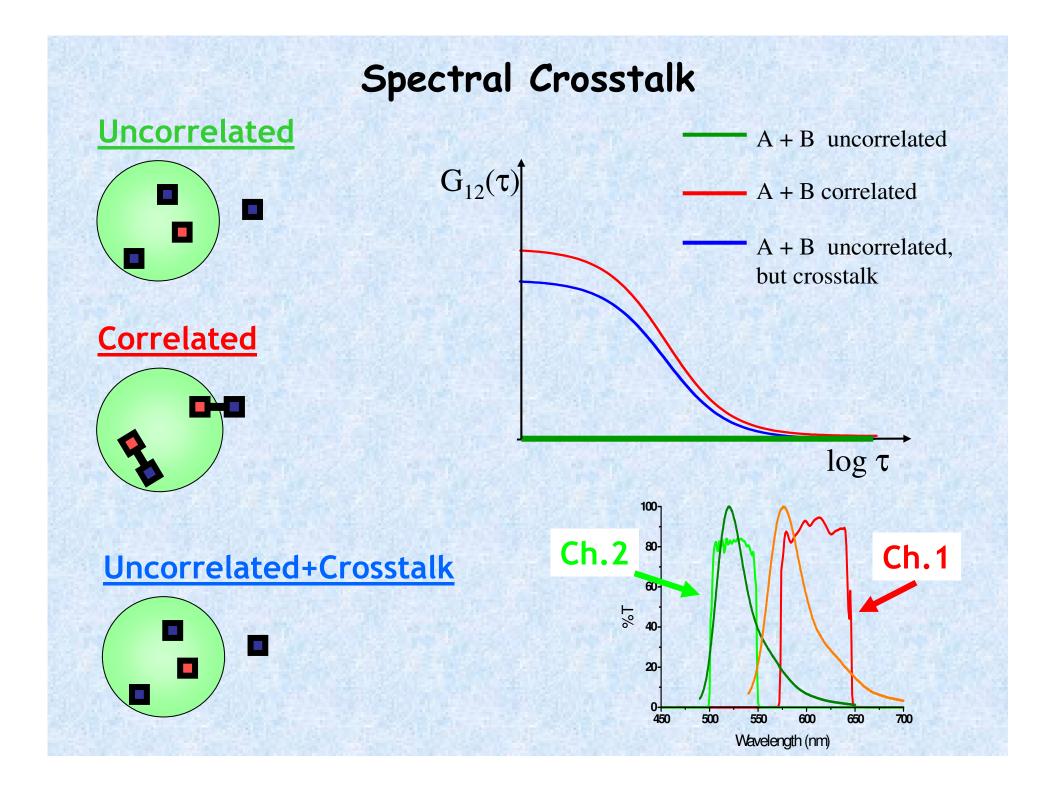
Autocorrelation from channel 2 on the **red particles**.

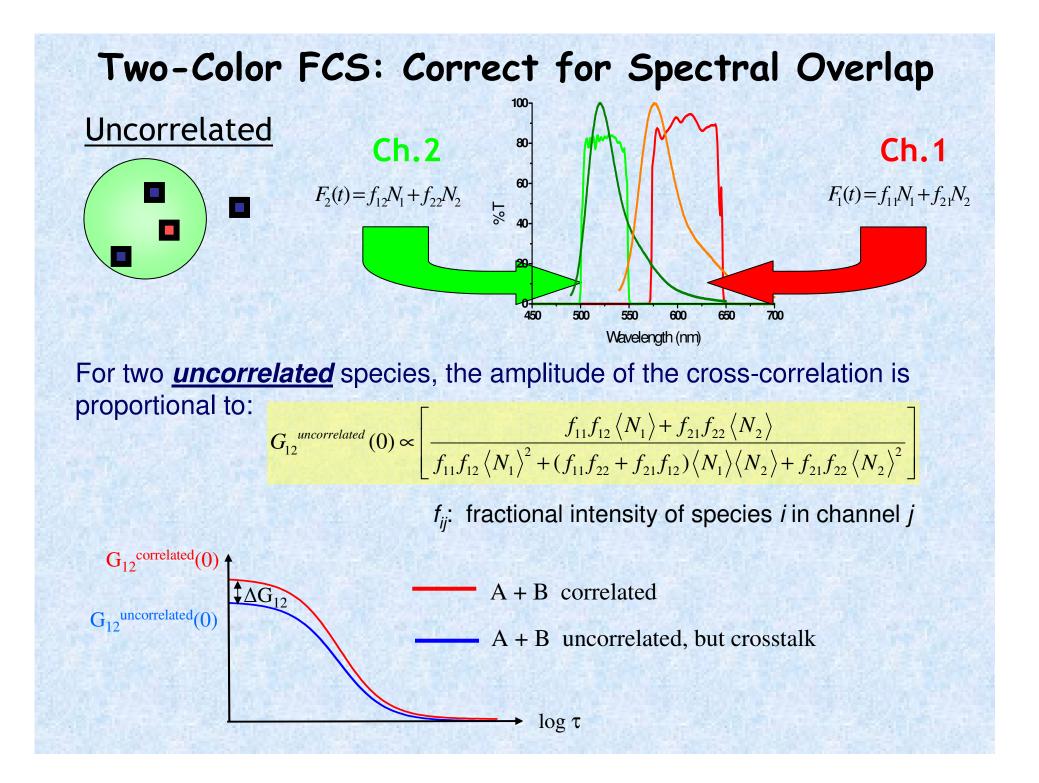
Signal

 $G_{12}(\tau)$

 $G_1(\tau)$

 $G_2(\tau)$

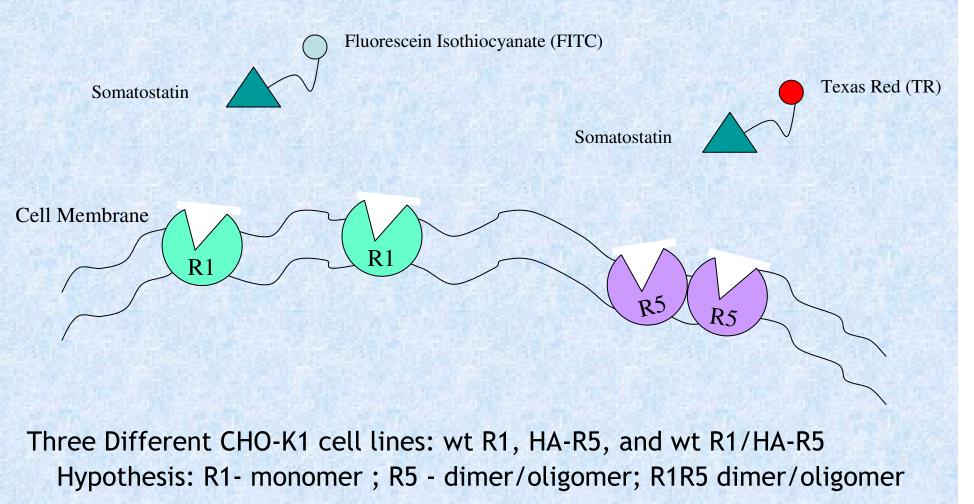




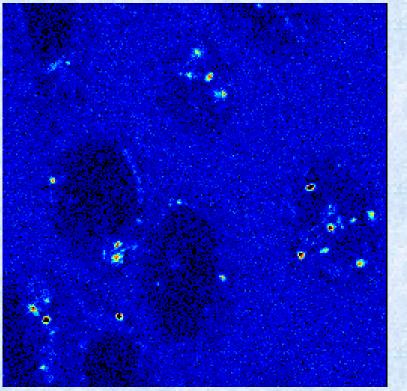
Does SSTR1 exist as a monomer after ligand binding while SSTR5 exists as a dimer/oligomer?

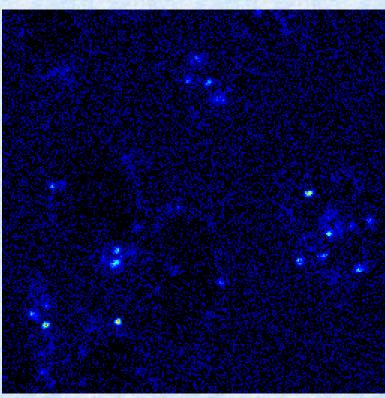
Collaboration with Ramesh Patel*† and Ujendra Kumar*

*Fraser Laboratories, Departments of Medicine, Pharmacology, and Therapeutics and Neurology and Neurosurgery, McGill University, and Royal Victoria Hospital, Montreal, QC, Canada H3A 1A1; †Department of Chemistry and Physics, Clarkson University, Potsdam, NY 13699

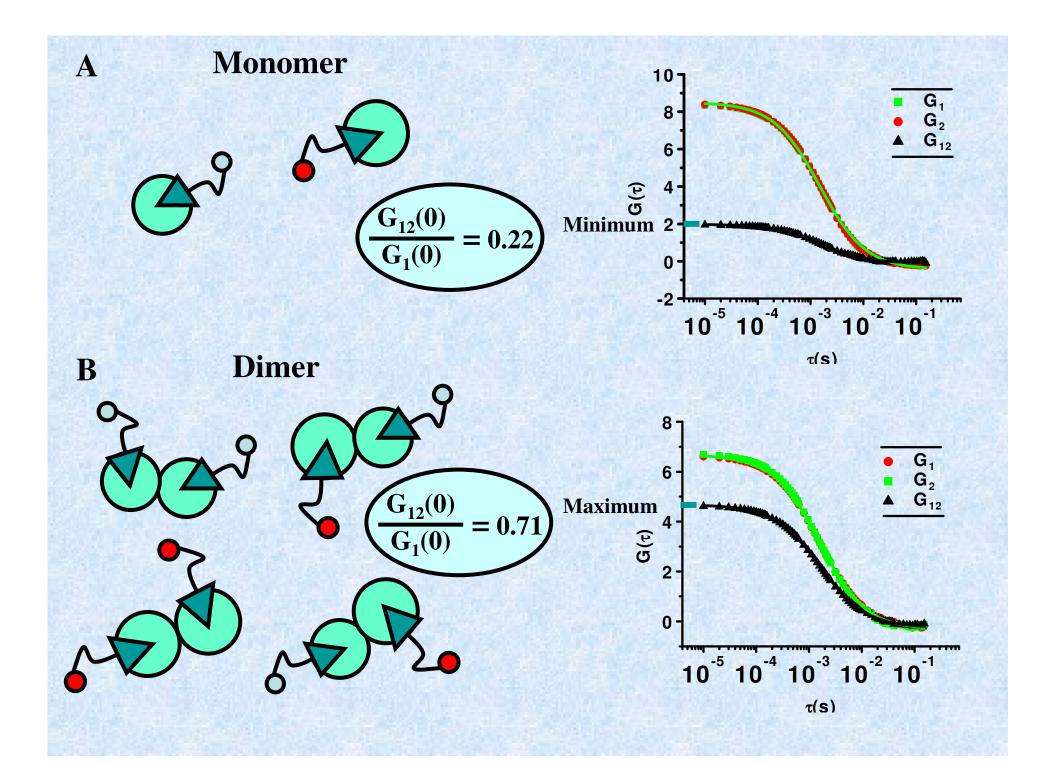


SSTR1 CHO-K1 cells with SST-fitc + SST-trGreen Ch.Red Ch.

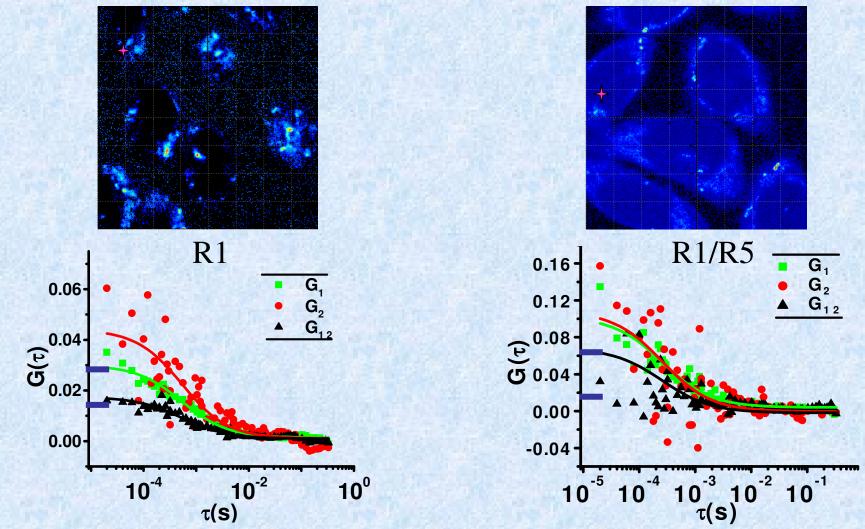




- Very little labeled SST inside cell nucleus
- Non-homogeneous distribution of SST
- Impossible to distinguish co-localization from molecular interaction



Experimentally derived auto- and cross-correlation curves from live R1 and R5/R1 expressing CHO-K1 cells using dual-color two-photon FCS.



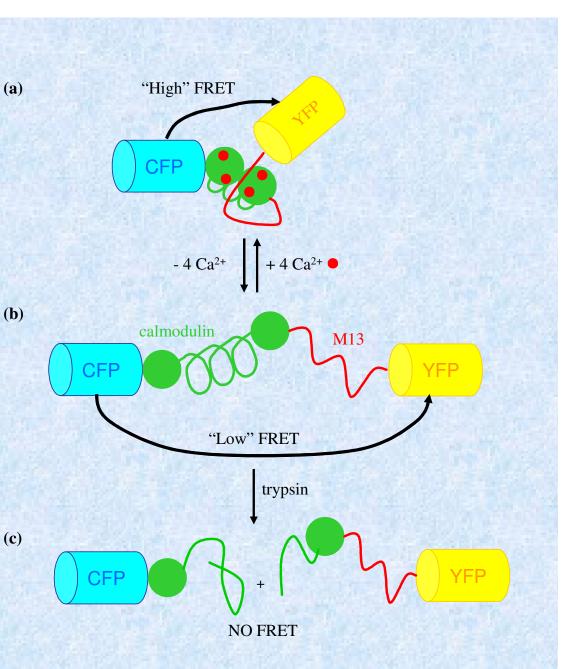
The R5/R1 expressing cells have a greater cross-correlation relative to the simulated boundaries than the R1 expressing cells, indicating a higher level of dimer/oligomer formation.

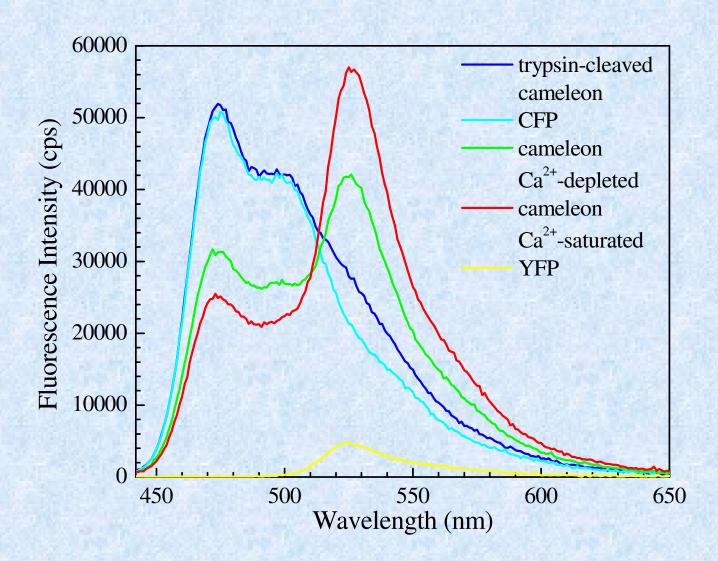
Patel, R.C., et al., Ligand binding to somatostatin receptors induces receptorspecific oligomer formation in live cells. PNAS, 2002. **99**(5): p. 3294-3299

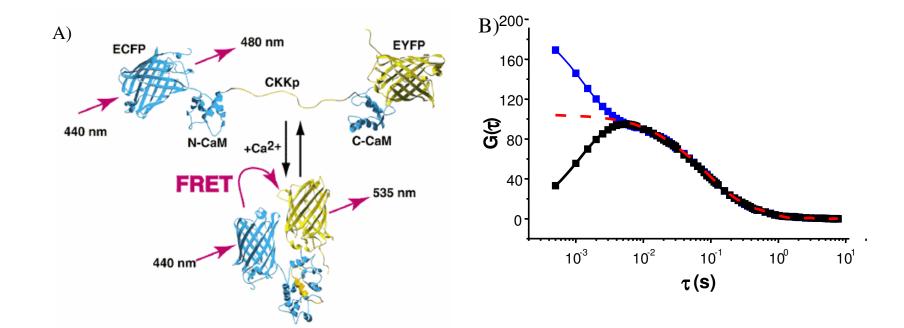
Molecular Dynamics

What if the distance/orientation is not constant?

- Fluorescence fluctuation can result from FRET or Quenching
- FCS can determine the rate at which this occurs
- This will yield hard to get information (in the µs to ms range) on the internal motion of biomolecules







. A) Cameleon fusion protein consisting of ECFP, calmodulin, and EYFP. [Truong, 2001 #1293] Calmodulin undergoes a conformational change that allows the ECFP/EYFP FRET pair to get cl ose enough for efficient energy transfer. Fluctuations between the folded and unfolded states will yield a measurable kinetic component for the cross-correlation. B) Simulation of how such a fluctuation would show up in the autocorrelation and cross-correlation. Red dashed line indicates pure diffusion.

In vitro Cameleon Data



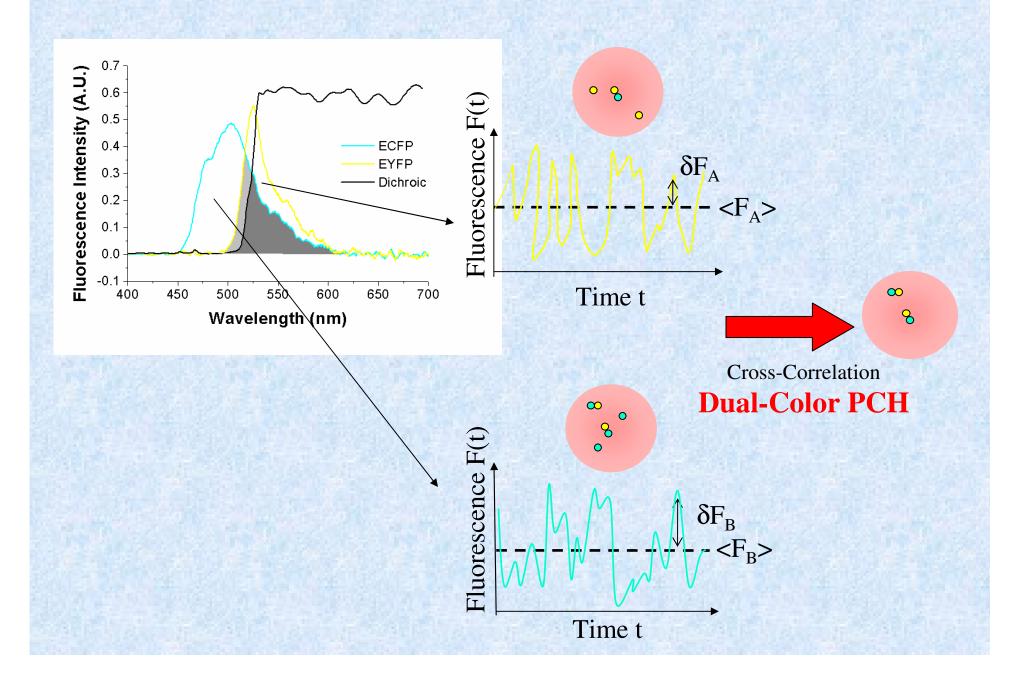
0.10-Donor Ch. Autocorrelation 0.08 Acceptor Ch. Autocorrelation Cross-correlation 0.06 0.04 **C**(म) 0.02 0.00 -0.02 -10⁻³ 10⁻⁵ 10⁻² 10⁻¹ 10[°] 10⁻⁴ τ (s)

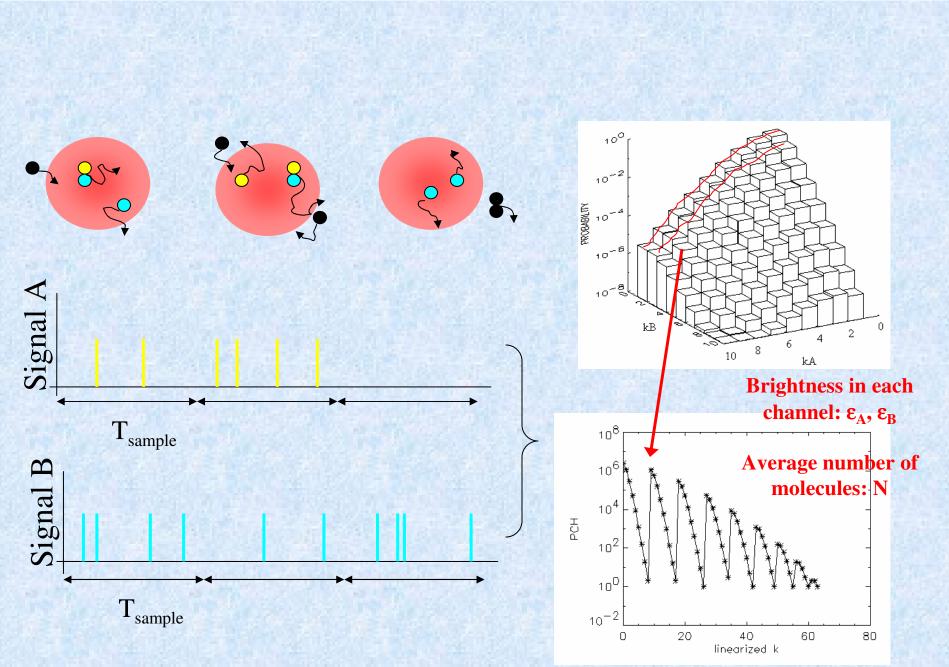
Ca²⁺ Saturated

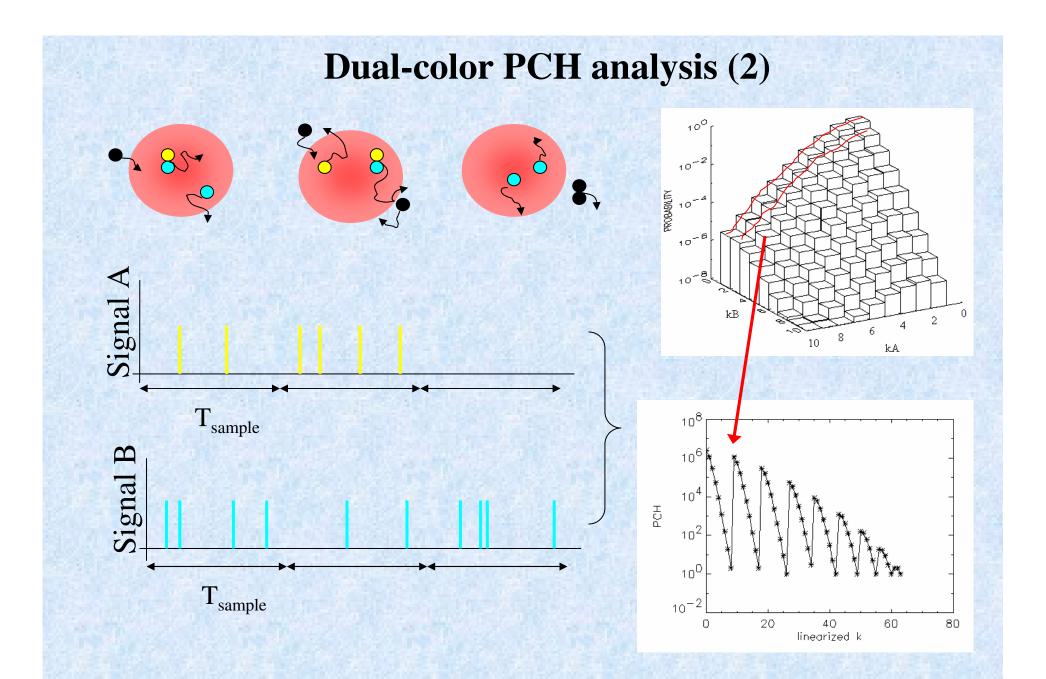
Crystallization And Preliminary X-Ray Analysis Of Two New Crystal Forms Of Calmodulin, B.Rupp, D.Marshak and S.Parkin, Acta Crystallogr. D 52, 411 (1996)

Are the fast kinetics (~20 μs) due to conformational changes or to fluorophore blinking?

Dual-color PCH analysis (1)



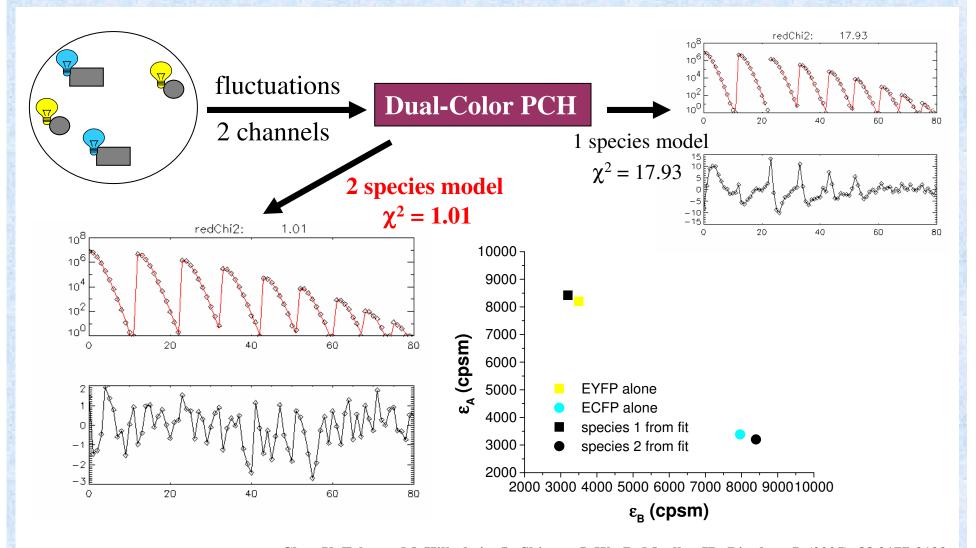




Single Species: $p(k_A, k_B) = PCH(\varepsilon_A, \varepsilon_B, N)$

Brightness in each channel: ε_A , ε_B Average number of molecules: *N*

Resolve Mixture of ECFP and EYFP in vitro



Chen Y, Tekmen M, Hillesheim L, Skinner J, Wu B, Mueller JD, Biophys. J. (2005), 88 2177-2192

ECFP & EYFP mixture resolved with single histogram.

Note: Cross-correlation analysis cannot resolve a mixture of ECFP & EYFP with a single measurement!